Physics in extra dimensions: lecture #1

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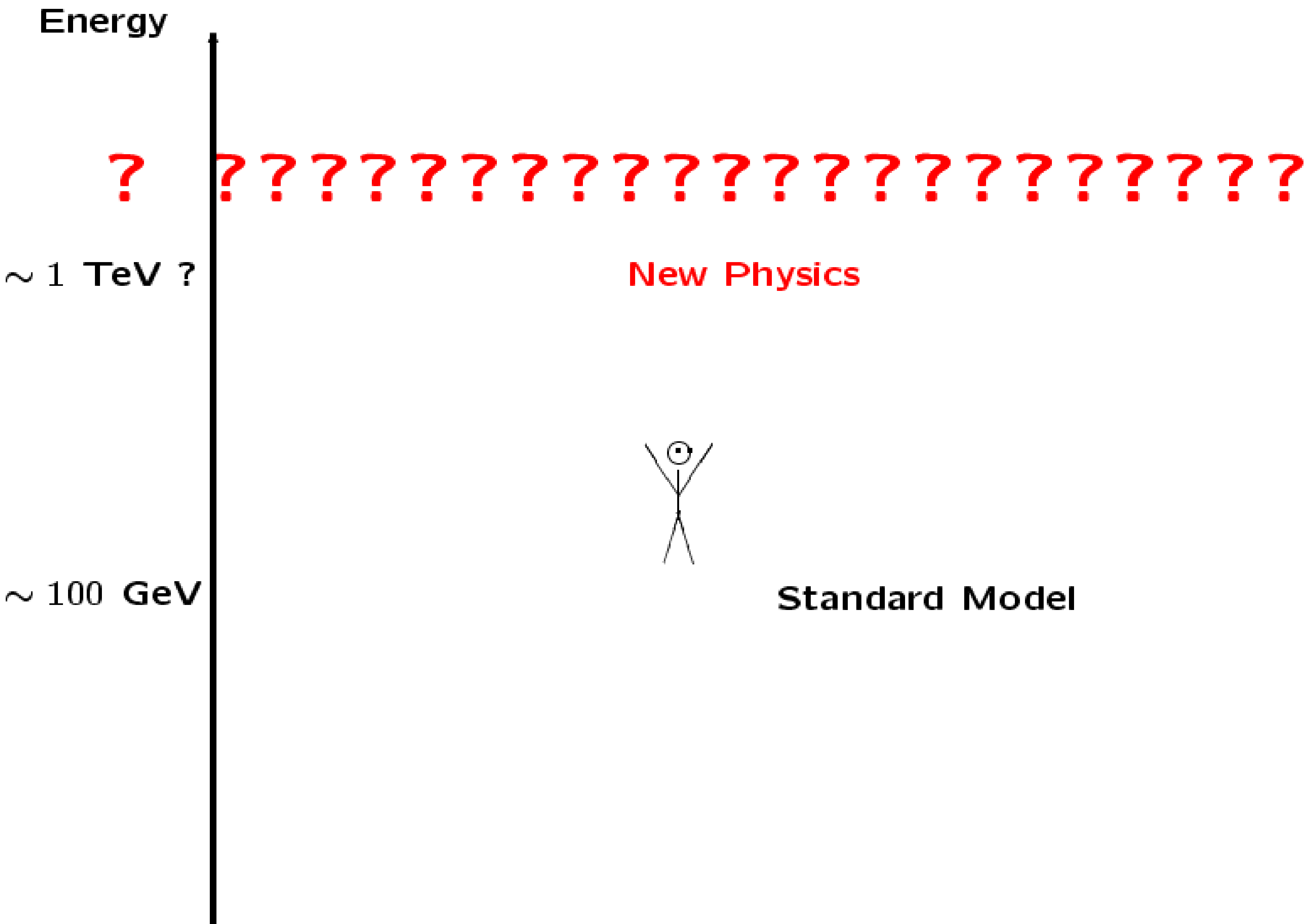
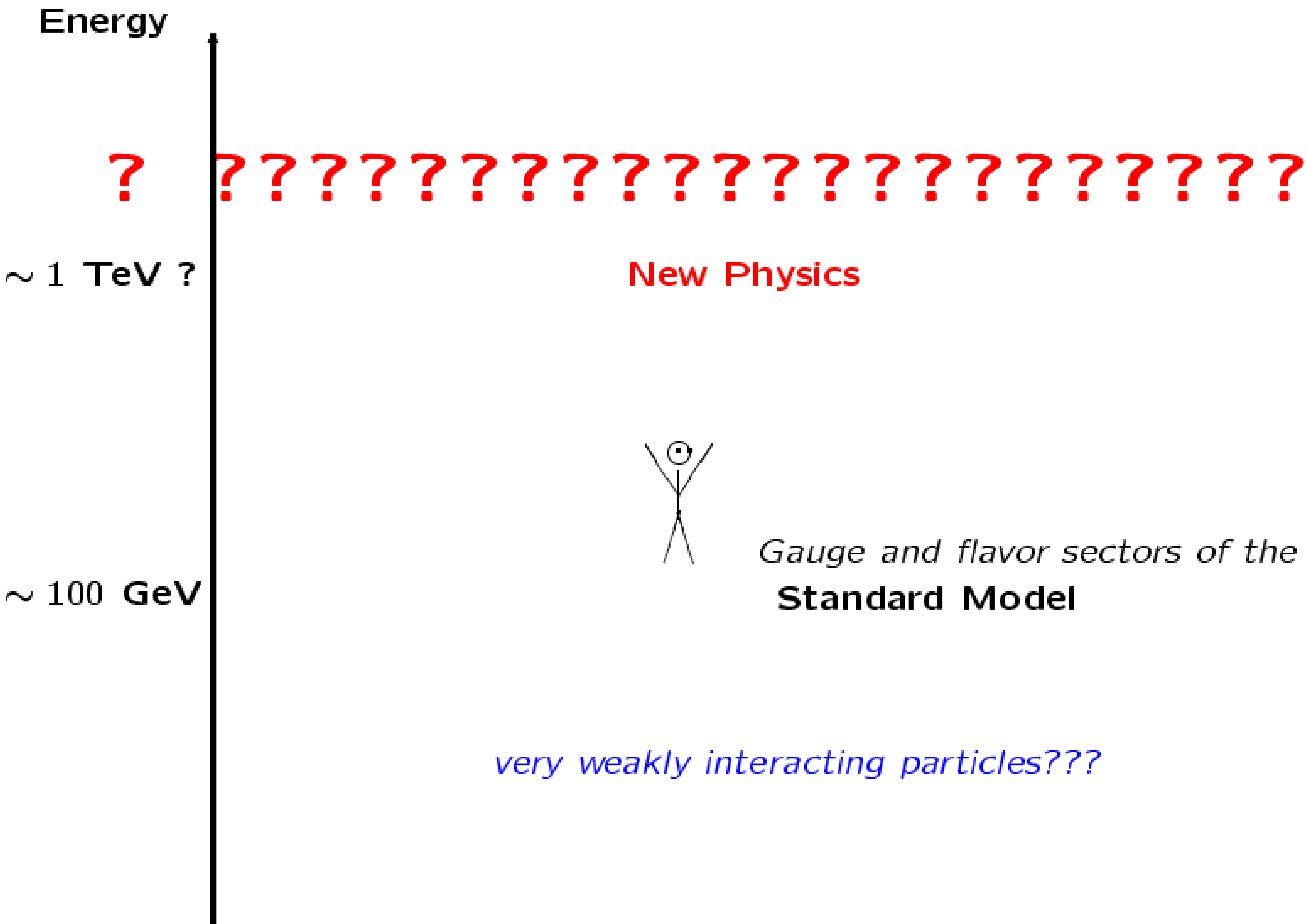
Lecture 1: **Field theory in compact dimensions.**
Gauge bosons in the bulk and their collider signatures.

Lecture 2: One universal extra dimension.
Discrete symmetries and cascade decays at colliders.

Lecture 3: Two universal extra dimensions.

Lecture 4: Particles in a warped extra dimension.


Energy
? ?????????????????????
~ 1 TeV ?
New Physics
~ 100 GeV
Standard Model



Energy
?
??????????????????????
~ 1 TeV ?
New Physics
~ 100 GeV
Gauge and flavor sectors of the
Standard Model
very weakly interacting particles???

“New physics” at the TeV scale could change the basic hypotheses of the Standard Model:

local quantum field theory
in 3 spatial + 1 time dimensions,
invariant under $SO(3,1)$ Lorentz transformations.

... *“terra incognita”* ... *“uncharted waters”* ...

Evidence that we live in 3 spatial dimensions:

- it is obvious! *(end of story?!?)*
- Gauss law, in 3 + n spatial dimensions: $V(r) \sim 1/r^{n+1}$
We observe $n = 0$ for gravity and electromagnetism.
- Standard Model agrees with the data.
- there are no renormalizable field theories in more dimensions

Counter-arguments:

- what's obvious may be due to preconception
(*e.g.*, quantum mechanics is not obvious)
- Gauss law may change at short distance
- Standard Model has not been tested below 10^{-16} cm
- gravitational interactions are non-renormalizable in $D = 3 + 1$

Types of extra dimensions:

- graviton only propagates in $n \geq 2$ flat extra dimensions (ADD)
- bosons only propagate in some flat extra dimensions (DDG)
- bosons and some fermions propagate in flat extra dimensions
- all particles propagate in some flat extra dimensions (UED)
- graviton only propagates in a warped extra dimension (RS)
- all particles propagate in a warped extra dimension

Bosons in compact spatial dimensions

4D flat spacetime $\perp$ one dimension of size $L = \pi R$:

A scalar field in the bulk, $\phi(x^\alpha)$:

$$\mathcal{L} = (\partial^\mu \phi)^\dagger \partial_\mu \phi - \left(\partial^4 \phi\right)^\dagger \partial_4 \phi - m_0^2 \phi^\dagger \phi \ , \qquad \mu = 0,1,2,3$$

$\Rightarrow$ **Equation of motion:** $\left(\partial^\mu \partial_\mu - \partial^4 \partial_4\right) \phi = m_0^2 \phi$

m_0 **is the 5D mass of ϕ.**

Neumann boundary conditions for “even” fields:

$$\frac{\partial}{\partial x^4}\phi(x^\mu, 0) = \frac{\partial}{\partial x^4}\phi(x^\mu, \pi R) = 0$$

Solution to the equation of motion:

$$\phi(x^\mu, x^4) = \frac{1}{\sqrt{\pi R}}\left[\phi^{(0)}(x^\mu) + \sqrt{2}\sum_{j\geq 1}\phi^{(j)}(x^\mu)\cos\left(\frac{jx^4}{R}\right)\right]$$

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Kaluza-Klein decomposition

Zero-mode (wave function is constant along x^4)

Kaluza-Klein modes: particles of definite momentum along x^4

4D point of view: a tower of massive particles:

$$m_j^2 = m_0^2 + \frac{j^2}{R^2}$$

Dirichlet boundary conditions for "odd" fields:

$$\phi(x,0) = \phi(x,\pi R) = 0$$

KK decomposition:

$$\phi(x^\mu, x^4) = \frac{\sqrt{2}}{\sqrt{\pi R}} \sum_{j\geq 1} \phi^{(j)}(x^\mu) \sin\left(\frac{j x^4}{R}\right)$$

There is no zero-mode.
The lightest KK mode is $\phi^{(1)}$, of mass $\sqrt{1/R^2 + m_0^2}$

Homework: *Check that the normalization condition for KK functions requires the factor of $\sqrt{2}$.*
Why $j < 0$ is not allowed?

Gauge bosons in 5D:

$A_\mu(x^\nu, x^4)$, $\mu, \nu = 0, 1, 2, 3$, and

$A_4(x^\nu, x^4)$ – polarization along the extra dimension.

From the point of view of the 4D theory:

$A_4(x^\nu, x^4)$ is a tower of spinless KK modes.

Gauge invariance requires A_μ to have a zero-mode:

$$\partial_4 A_\mu(x^\nu, 0) = \partial_4 A_\mu(x^\nu, \pi R) = 0$$

$$A_\mu(x^\nu, x^4) = \frac{1}{\sqrt{\pi R}} \left[A_\mu^{(0)}(x^\nu) + \sqrt{2} \sum_{j \geq 1} A_\mu^{(j)}(x^\nu) \cos\left(\frac{j x^4}{R}\right) \right]$$

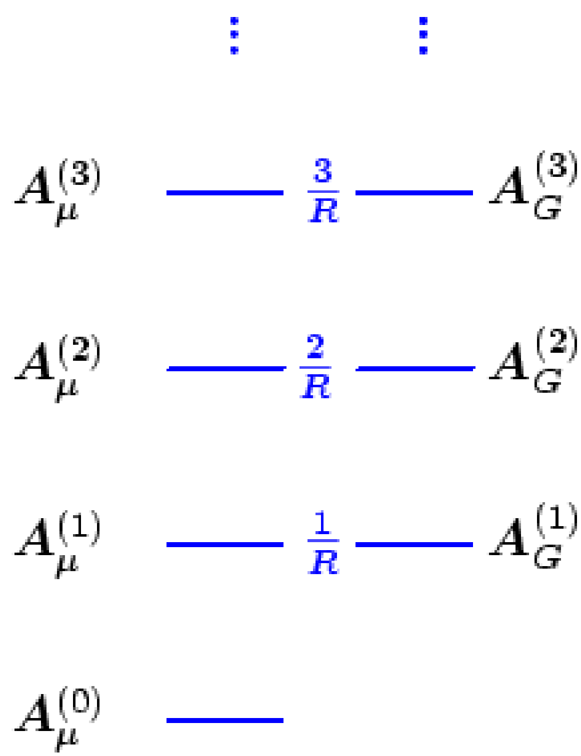
Dirichlet B.C : $A_4(x^\nu, 0) = A_4(x^\nu, \pi R) = 0$

KK decomposition : $A_4(x^\nu, x^4) = \sqrt{\frac{2}{L}} \sum_{j \geq 1} A_G^{(j)}(x) \sin\left(\frac{j x^4}{R}\right)$

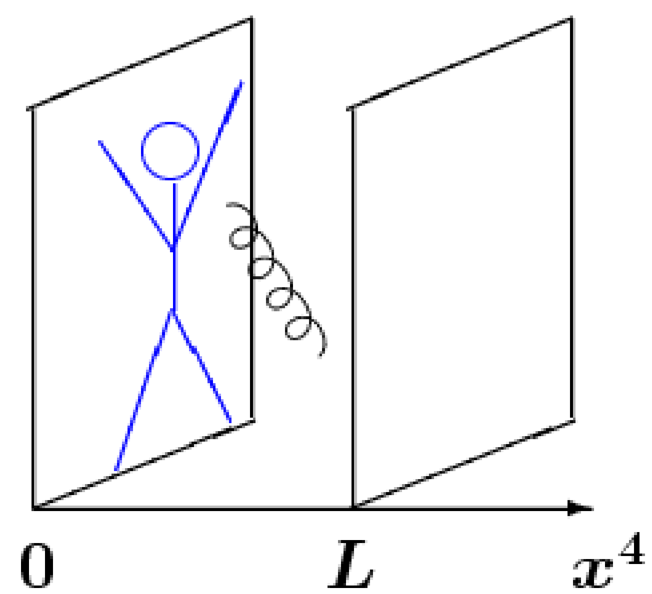
$\rightarrow$ $A_4(x^\nu, x^4)$ **does not have a 0-mode!** *(Odd field)*

Kaluza-Klein spectrum of gauge bosons

$A_G^{(j)}(x^\nu)$ becomes the longitudinal degree of freedom of the spin-1 KK mode $A_\mu^{(j)}(x^\nu)$.



Assume that only bosons propagate in a flat extra dimension, and that the fermions are localized at $x^4 = 0$.

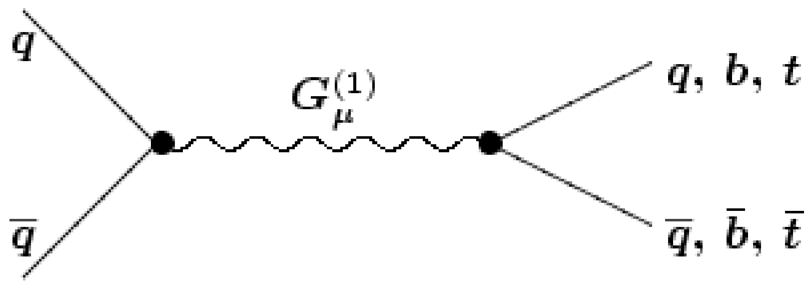


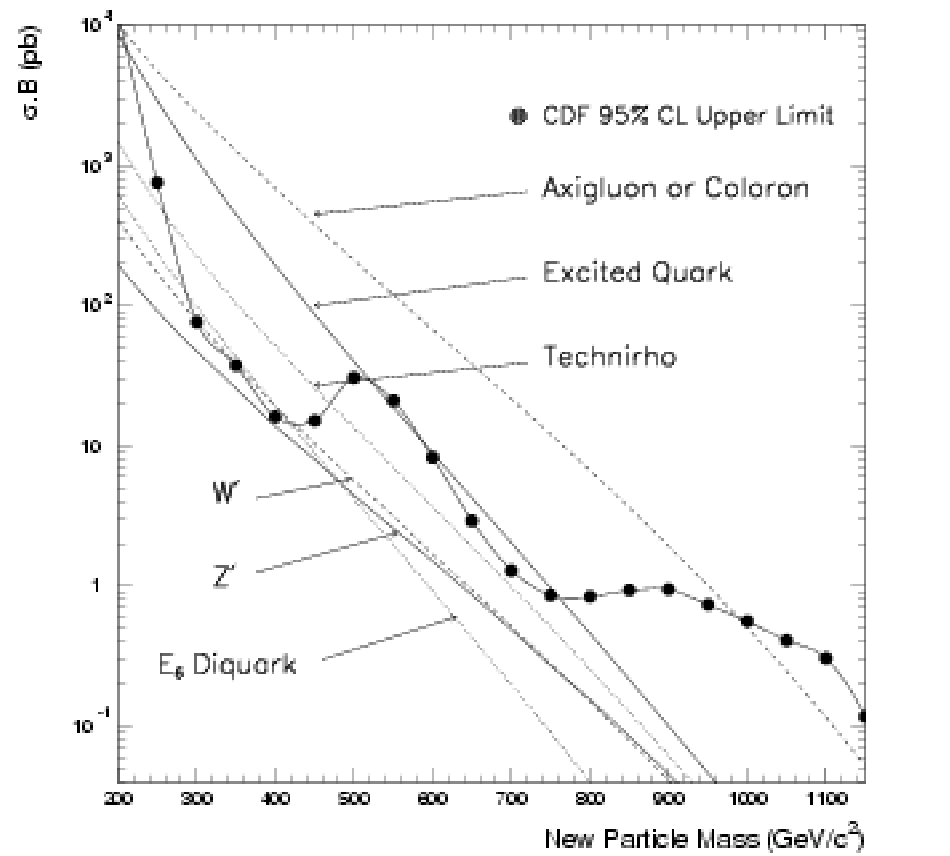
Interactions of the KK gluons with quarks:

$$
\begin{aligned}
\mathcal{L}_{4D} &= \int_0^L dx^4 \, g_5 G^a_\mu(x^\nu, x^4) \left[\delta\left(x^4\right) \bar{q}(x^\nu)\gamma^\mu T^a q(x^\nu) \right] \\
&= g_s \left(G^{(0)a}_\mu + \sqrt{2} \sum_{j \geq 1} G^{(j)a}_\mu(x) \right) \bar{q}\gamma^\mu T^a q
\end{aligned}
$$

KK gluon production (in the narrow width approximation):

$$\sigma\left(p\bar{p} \to G_\mu^{(1)} X\right) \approx \frac{16\pi^2\alpha_s}{9s} \sum_q \int_{M^2/s}^1 \frac{dx}{x} \left[q(x)\, q\left(\frac{M^2}{xs}\right) + \overline{q}(x)\, \overline{q}\left(\frac{M^2}{xs}\right) \right]$$





Run I limit on dijet resonances:

$\longrightarrow \quad 1/R > 1.1$ TeV

5D theory = 4D theory with some heavy particles

$SU(3)_c$ in extra dimensions $\rightarrow$ SM gluon + heavy gluons

4D theory with the same spectrum must include a larger gauge symmetry.

$SU(3)_1 \times SU(3)_2 \rightarrow SU(3)_c$ spontaneously broken by the VEV of a scalar transforming as $(3, \bar{3})$

Quarks transform as 3 of $SU(3)_1$

G^a_μ - massless gluon as in QCD, with $g_s = \frac{h_1 h_2}{\sqrt{h_1^2+h_2^2}}$

G'^a_μ - massive gluon ("coloron") with couplings $g_s \frac{h_1}{h_2} G'^a_\mu \bar{q}\gamma^\mu T^a q$

KK gluon coupling recovered if the two $SU(3)$ gauge couplings satisfy $h_1/h_2 = \sqrt{2}$.

The 4D theory describing the first N KK modes of the gluon has a $SU(3)_1 \times SU(3)_2 \times \cdots \times SU(3)_{N+1} \to SU(3)_c$ gauge structure.

Assume that the Higgs doublet and electroweak gauge bosons propagate in one flat extra dimension, while the fermions are localized at one end of the interval.

Homework: Derive the couplings of the KK electroweak gauge bosons to quark and leptons.

LEP-II limits on four-fermion interactions imply $1/R > 6$ TeV.

Fermions

All Standard Model fermions are chiral.

The two top quarks:

- "left-handed" top *(feels the weak interaction)*
- "right-handed" top *(no interaction with $W^{\pm}$)*

$$\begin{pmatrix} t_L \\ b_L \end{pmatrix} \qquad \boxed{t_R}$$

Fermions

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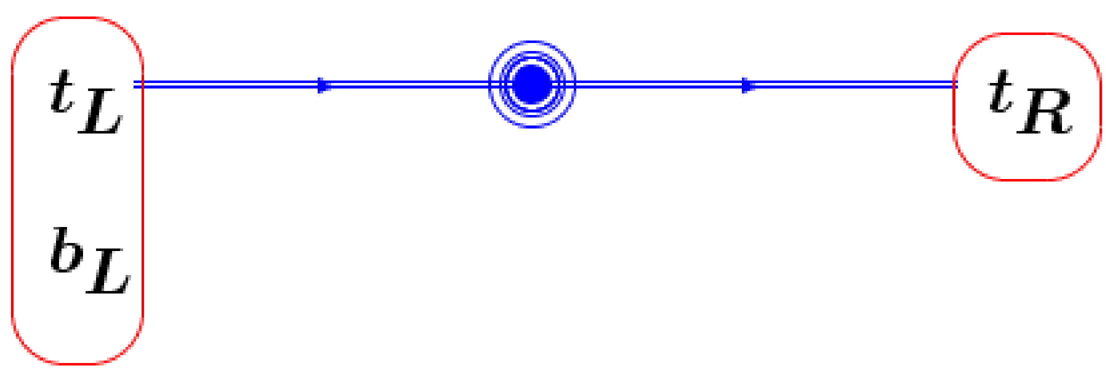
The two top quarks:

- **"left-handed" top** *(feels the weak interaction)*
- **"right-handed" top** *(no interaction with* $W^{\pm}$*)*

Top mass: t_L **turns into** t_R **and vice-versa**



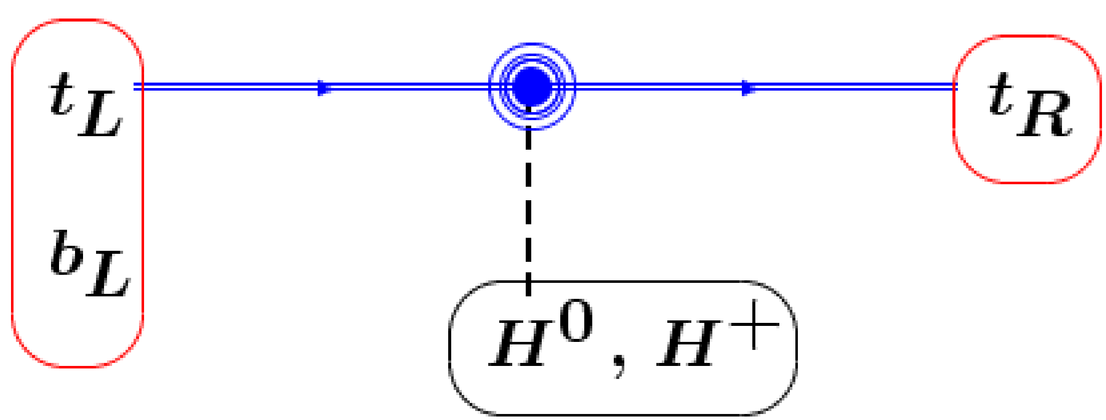
Fermions

All Standard Model fermions are chiral.

Top quark gets a mass from its interaction with the vacuum:

$\lambda_t \, \bar{t}_R \langle H^0 \rangle t_L$, $\qquad \langle H^0 \rangle \approx 174$ GeV

Measured top mass $\Rightarrow$ coupling constant is $\lambda_t \approx 1$.



Fermions in a compact dimension

Lorentz group in 5D $\Rightarrow$ vector-like fermions:

$$\chi = \chi_L + \chi_R$$

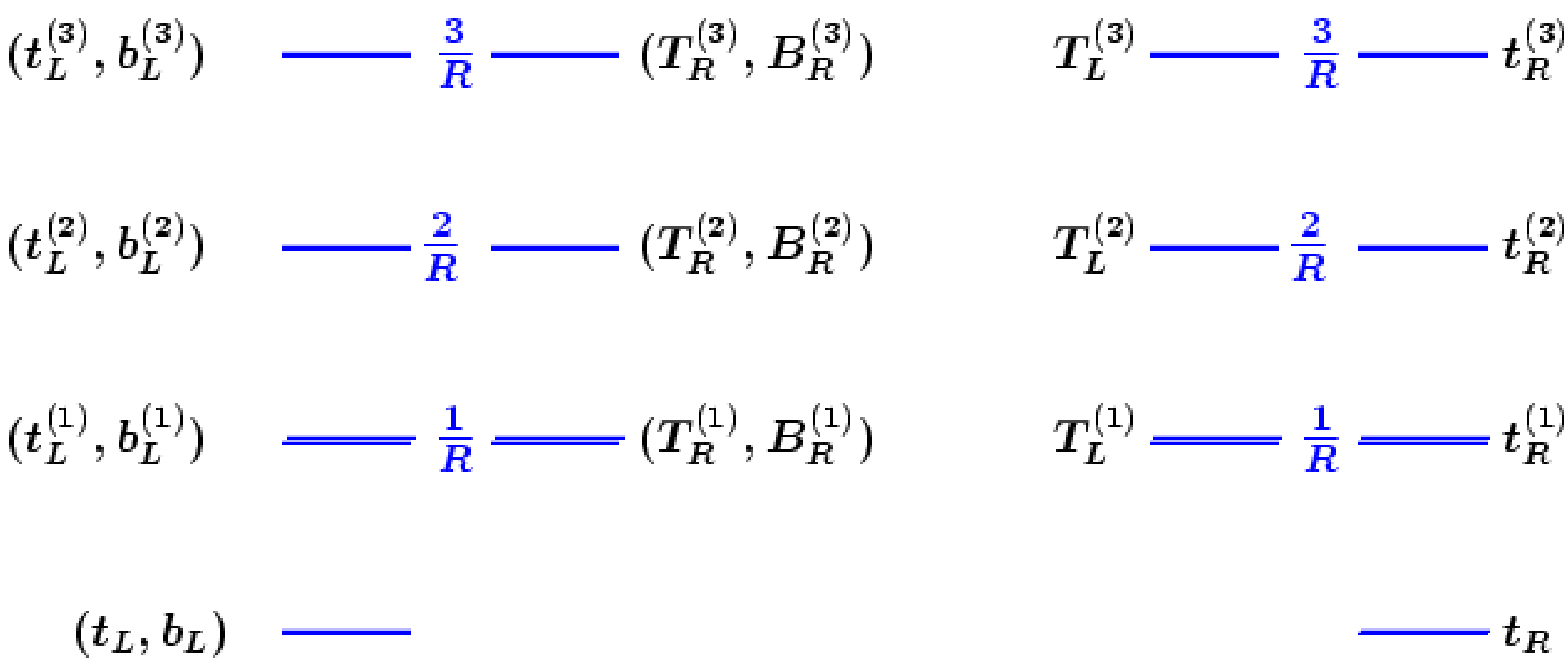
Chiral boundary conditions:

$$\begin{aligned} \chi_L(x^\mu, 0) &= \chi_L(x^\mu, \pi R) = 0 \\ \frac{\partial}{\partial x^4}\chi_R(x^\mu, 0) &= \frac{\partial}{\partial x^4}\chi_R(x^\mu, \pi R) = 0 \end{aligned}$$

Kaluza-Klein decomposition:

$$\chi = \frac{1}{\sqrt{\pi R}}\left\{\chi_R^0(x^\mu) + \sqrt{2}\sum_{j\geq 1}\left[\chi_R^j(x^\mu)\cos\left(\frac{\pi j x^4}{L}\right) + \chi_L^j(x^\mu)\sin\left(\frac{\pi j x^4}{L}\right)\right]\right\}$$

Kaluza-Klein spectrum of quarks and leptons



G. D. Kribs, TASI lectures on the
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E. H. Simmons, "Coloron phenomenology,"
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K. R. Dienes, E. Dudas and T. Gherghetta, "Grand unification at intermediate mass scales through extra dimensions,"
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...

Conclusions so far

- Extra spatial dimensions may exist if their size is small enough. The limits are model dependent.

- Any particle that propagates in $D \geq 5$ would appear in experiments as a tower of heavy 4-dimensional particles.

- If bosons propagate in extra dimensions while fermions are localized, then the Kaluza-Klein bosons may be singly produced, leading to s-channel resonances.

- Kaluza-Klein modes of the quarks and leptons are <u>vectorlike</u> fermions.